

**Exercise 23**

Differentiate.

$$f(x) = \frac{x^2 e^x}{x^2 + e^x}$$

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**Solution**Use the quotient rule and the product rule to differentiate  $f(x)$ .

$$\begin{aligned} f'(x) &= \frac{d}{dx} \left( \frac{x^2 e^x}{x^2 + e^x} \right) \\ &= \frac{\left[ \frac{d}{dx}(x^2 e^x) \right] (x^2 + e^x) - \left[ \frac{d}{dx}(x^2 + e^x) \right] (x^2 e^x)}{(x^2 + e^x)^2} \\ &= \frac{\left\{ \left[ \frac{d}{dx}(x^2) \right] e^x + x^2 \left[ \frac{d}{dx}(e^x) \right] \right\} (x^2 + e^x) - (2x + e^x)(x^2 e^x)}{(x^2 + e^x)^2} \\ &= \frac{[(2x)e^x + x^2(e^x)](x^2 + e^x) - (2x + e^x)(x^2 e^x)}{(x^2 + e^x)^2} \\ &= \frac{2xe^{2x} + x^4 e^x}{(x^2 + e^x)^2} \\ &= \frac{xe^x(2e^x + x^3)}{(x^2 + e^x)^2} \end{aligned}$$